Chapter 1

# MEASUREMENTS

## LEARNING OBJECTIVES

**At the end of this chapter the students will be able to:**

- ★ Understand what is Physics.
- ★ Understand that all physical quantities consist of a numerical magnitude and a unit.
- ★ Describe and use base units, supplementary units, and derived units.
- ★ Understand and use the scientific notation.
- ★ Use the standard prefixes and their symbols to indicate decimal sub-multiplies or multiples to both base and derived units.
- ★ Understand and use the conventions for indicating units.
- ★ Understand the distinction between systematic errors and random errors.
- ★ Understand and use the significant figures.
- ★ Understand the distinction between precision and accuracy.
- ★ Use dimensionality to check the homogeneity of physical equations.
- ★ Derive formulae in simple cases using dimensions.

## INTRODUCTION TO PHYSICS

Man has always been curious to know about things. He started to observe, think and wondering about the world around him. He tried to find ways to organize the disorder in observed facts about natural phenomena and material objects things in orderly manner. His attempts resulted in the birth of a single discipline (Branch) of science, called natural philosophy.

There was a huge increase in the volume of scientific knowledge up till the beginning of nineteenth century and it was found necessary to classify the study of nature into two branch.

(i) Biological Sciences.

(ii) Physical Science.

### Biological Sciences

The science which deals with living things such as botany, zoology etc are called biological sciences.

### Physical Sciences

The science which deals with non living things such as chemistry, astronomy, geology etc are called physical sciences.

Physics is important and basic part of physical science besides its other disciplines such as chemistry, astronomy geology etc. Physics is an experimental science and scientific method emphasizes the need of accurate measurement of different phenomena or of man made objects

**Areas of Physics**

Mechanics
Heat & thermodynamics
Electromagnetism
Optics
Sound
Hydrodynamics
Special relativity
General relativity
Quantum mechanics
Atomic physics
Molecular physics
Nuclear physics
Solid-state physics
Particle physics
Superconductivity
Super fluidity
Plasma physics
Magneto hydrodynamics
Space physics

## Frontiers of Fundamental Science

At the present time there are three main frontiers of fundamental science.

(1) The world of the extremely large, the universe itself, Radio telescopes now gather information from the far side of the universe and have recently detected, as radio waves. the "fire light" of the big bang which probably started off the expanding universe nearly 20 billion years ago.

(2) The world of extremely small, that of the particles such as, electron, protons, neutrons, mesons and others.

(3) The world of complex matter, it is also the world of "middle sized" things, from molecules at one extreme to the earth at the other. This is all fundamental physics, which is heart of science.

### *Q.1 Define physics. Give its main branches.*

**Ans.** **PHYSICS**

***"The branch of science which deals with the study of matter and energy and the relationship between them is called physics."***

The study of physics involves investigating of such things as the laws of motion, the structure of space and time, the nature and type of forces that hold different materials together, the interaction between different particles, the interaction of electromagnetic radiation with matter and so on.

**Interdisciplinary Areas of Physics**

- Astrophysics
- Biophysics
- Chemical physics
- Engineering physics
- Geophysics
- Medical physics
- Physical oceanography
- Physics of music

## Branches of Physics

By the end of 19th century many physicists started believing that everything about physics has been discovered. However, about the beginning of the 20th century many new experimental facts showed that the laws formulated by the previous scientists need modifications. Further researches gave birth to many new disciplines (branches).

**(1) Nuclear Physics**

The branch of physics which deals with atomic nuclei, is called nuclear physics.

**(2) Particle Physics**

The branch of physics which is concerned with the ultimate particles of which matter is composed of is called particle physics.

**(3) Relativistic Mechanics**

The branch of physics which deals with velocities approaching that of light is called relativistic mechanics.

**(4) Solid State Physics**

The branch of physics which is concerned with the structure and properties of solid materials is called solid state physics.

**Do You Know?**

Computer chips are made from wafers of the metalloid silicon, a semiconductor.

## Other Branches of Science

Physics is the most fundamental of all sciences and provides other branches of science, basic principles and fundamental laws. This overlapping of physics and other fields gave birth to new branches such as physical Chemistry, biophysics, astrophysics, health physics etc.



### *Q.2 What is the role of physics in technology?*

**Ans.** **ROLE OF PHYSICS IN TECHNOLOGY**

Physics also plays an important role in development of technology and engineering.

Science and technology are potent force for the change in the outlook of mankind. The information media and fast means of communications have brought all parts of the world in close contact with one another. Events in one part of the world are immediately reverberate round the globe.

We are living in the age of information technology. The computer networks are products of chips developed from basic ideas of physics. The chips are made of silicon. Silicon can be obtained from sand. It is upto us whether we make a sand castle or computer out of it.

### *Q.3 What do you mean by physical quantities? Also describe its types.*

**Ans.** **PHYSICAL QUANTITIES**

The foundation of physics rests upon physical quantities in terms of which the laws of physics one expressed. Therefore the qualities have to be measured accurately.

All those quantities which can be measured are called physical quantities.

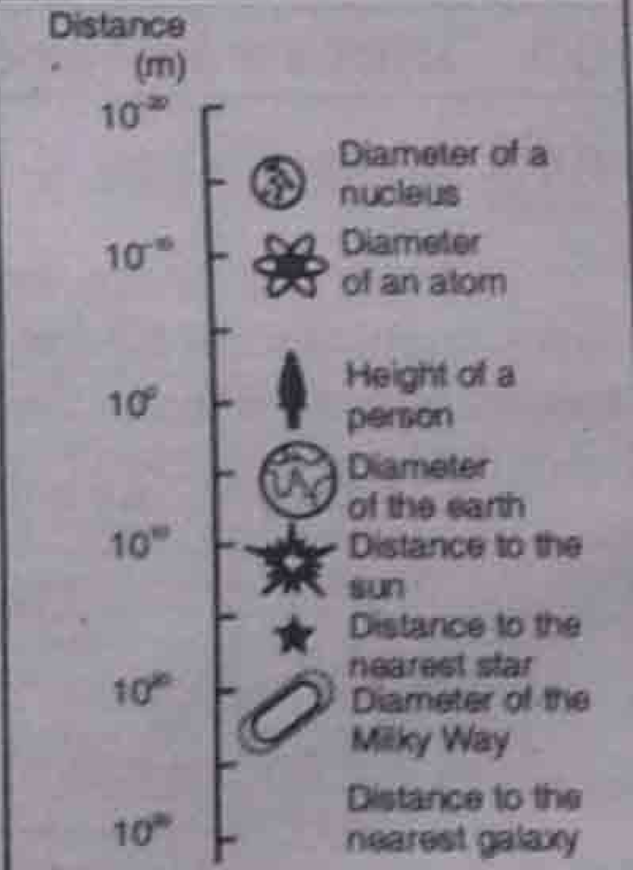
e.g., mass length, time, velocity, force, density temperature, electric current, volume, acceleration etc.

Physical quantities have been divided into two categories.

(i) Base quantities.

(ii) Derived quantities.

**Do You Know?**




**Order of magnitude of some distances**

**(i) Base Quantities**

Base quantities are those quantities which cannot be defined in terms of other physical quantities. These are the minimum number of physical quantities in term of which other physical quantities can be defined. The typical examples of base quantities are length, mass and time.

**(ii) Derived Quantities**

Those physical quantities whose definitions are expressed in terms of other physical quantities are called derived quantities.

The examples of derived quantities are velocity, acceleration, force, momentum etc.

## Measurement of Base Quantities

The measurement of base quantity is based on two steps.

(i) The choice of standard.

(ii) The establishment of a procedure for comparing the quantity to be measured with the standard so that number and a unit are determined as the measure of that quantity.

## Characteristics of an Ideal Standard

An ideal standard has two principal characteristics: It is accessible and it is invariable. These two requirements are often incompatible and a compromise has to be made between them.

**Unit**

To measure a physical quantity, a standard size of that quantity is required. This standard size is known as unit for that particular physical quantity.

### *Q.4 What is international system of units?*

**Ans.** INTERNATIONAL SYSTEM OF UNITS

In 1960 an international committee agreed on a set of definitions and standard to describe the physical quantities. The system that was established is called system international (SI). Since then SI units are being used by world's scientific community in all scientific works. The international system of units (SI) is built up from three kinds of units.

(1) Base units (2) Supplementary units (3) Derived units

### *Q.5 What are base units? Define the base units of SI.*

**Ans.** BASE UNITS

There are seven base units for various physical quantities namely; length, mass, time, temperature, electric current, luminous intensity and amount of substance (with special reference to the number of particles).

The name of base units for these physical quantities together with symbols are listed in table:

| Physical Quantity | SI Unit | Symbol |
|---|---|---|
| Length | Metre | m |
| Mass | Kilogram | kg |
| Time | Second | s |
| Electric current | Ampere | A |
| Thermodynamic temperature | Kelvin | K |
| Intensity of light | Candela | cd |
| Amount of substance | Mole | mol |

### *Q.6 What are supplementary units? Define them.*

**Ans.** SUPPLEMENTARY UNITS

The general conference on weights and measures has not yet classified certain units of the SI under either base units or derived units. These SI units are called supplementary units. This class contains two units, which are:

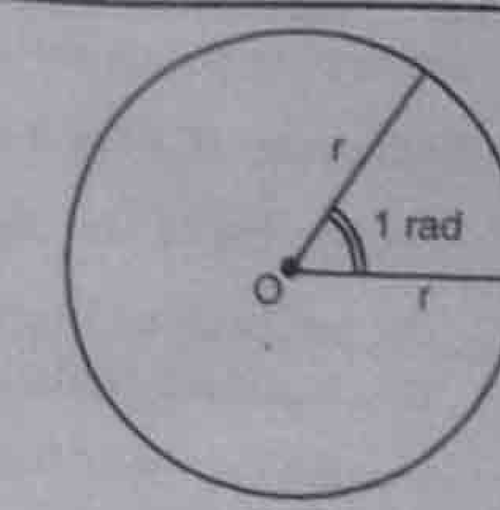
(1) Radian (Plane angle)

(2) Steradian (Solid angle)



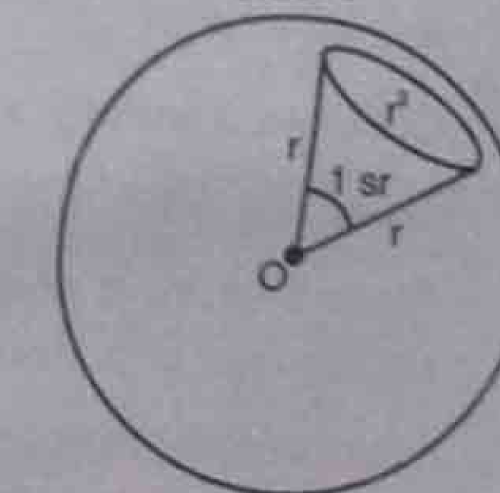
## Radian

The radian is the plane angle between two radii of a circle which cut off on the circumference an arc, equal in length to the radius, as shown in figure.




## Steradian

The steradian is the solid angle (three dimensional angle) subtended at the centre of the sphere by an area of its surface equal to the square of radius of the sphere, as shown in figure.




### *Q.7 What are derived units? Give some examples.*

**Ans.** DERIVED UNITS

SI units for measuring all other physical quantities are derived from the base and supplementary units. Some of the derived units are given below:

| Physical Quantity | Unit | Symbol | In terms of base units |
|---|---|---|---|
| Force | Newton | N | $kg\ ms^{-2}$ |
| Work | Joule | J | $N\ m = kg\ m^2\ s^{-2}$ |
| Power | Watt | W | $Js^{-1} = kg\ m^2\ s^{-3}$ |
| Pressure | Pascal | Pa | $Nm^{-2} = kg\ m^{-1}\ s^{-2}$ |
| Electric charge | Coulomb | C | A s |

### *Q.8 What do you understand by term?*

**Ans.** SCIENTIFIC NOTATION

Numbers are expressed in standard form called scientific notation, which employs power of ten. The internationally accepted practice is that there should be only one non-zero digit left of decimal.

**Scientific Notation Explain the Uses of Prefix**

*Example*

The number 134.7 should be written as

$$134.7 = 1.374 \times 10^2$$

Similarly, the number 0.0023 can be written as

$$0.0023 = 2.3 \times 10^{-3}$$

## Conventions for Indicating Units

Use of SI units requires special care, more particularly in writing prefixes.

**Some Prefixes for Powers of Ten**

| Factor | Prefix | Symbol |
|---|---|---|
| $10^{-18}$ | atto | a |
| $10^{-15}$ | femto | f |
| $10^{-12}$ | pico | p |
| $10^{-9}$ | nano | n |
| $10^{-6}$ | micro | μ |
| $10^{-3}$ | mili | m |
| $10^{-2}$ | centi | c |
| $10^{-1}$ | deci | d |
| $10^{1}$ | deca | da |
| $10^{3}$ | kilo | k |
| $10^{6}$ | mega | M |
| $10^{9}$ | giga | G |
| $10^{12}$ | tera | T |
| $10^{15}$ | peta | P |
| $10^{18}$ | exa | E |

Following points should be kept in mind while using units:

(i) Full name of the unit does not begin with a capital even if named after a scientist e.g. newton.

(ii) The symbol of unit named after a scientist has initial capital letters such as N for newton.

(iii) The prefix should be written before the unit without any space, such as $1 \times 10^{-3}$, m is written as 1 mm. Standard prefixes are given in table 1.4.

(iv) A combination of base units is written each with one space apart. For example, newton metre is written as N m.

(v) Compound prefixes are not allowed. For example, 1μμF may be written as 1pF.

(vi) A number such as $5.4 \times 10^{4}$ cm may be expressed in scientific notation as $5.0 \times 10^{2}$m.

(vii) When a multiple of a base unit is raised to a power, the power applies to the whole multiple and not the base unit alone. Thus, $1\text{km}^2 = 1\ (\text{km})^2 = 1 \times 10^6\text{m}^2$.

(viii) Measurement in practical work should be recorded immediately in the most convenient unit, e.g., micrometer screw guage measurement in mm, and the mass of calorimeter in grams (g). But before calculation for the result, all measurements must be converted to the appropriate SI base units.

### Q.9 *Explain the phenomenon of errors and uncertainties.*

**Ans.** ERRORS AND UNCERTAINTIES

All physical measurements are uncertain or imprecise to some extent. It is very difficult to eliminate all possible errors or uncertainties in a measurement. The errors in a measurement may occur due to.

(1) Negligence or inexperience of a person.

(2) The faulty apparatus.

(3) Inappropriate method or technique.

The uncertainty may occur due to inadequacy or limitations of an instrument, natural variations of the object being measured or natural imperfection of a person's senses. However the uncertainty is also usually described as an error in a measurement.

#### Types of Errors

There are two types of errors.

(1) Random error.

(2) Systematic error.

**(1) Random Error**

Random error is said to occur when repeated measurements of a quantity, give different values under the same conditions. It is due to some unknown causes.

Repeating the measurements several times and taking an average can reduce the effect of random errors.

**(2) Systematic Errors**

*Systematic error refers to an effect that influences all measurements of a particular quantity equally. It produces consistent difference in reading.*



It occurs due to:

(i) Zero error of instrument.

(ii) Poor calibration of instruments, or incorrect marking etc.

Systematic error can be reduced by comparing the instrument with an other which is known to be more accurate. Thus for systematic error, a correction factor can be applied to reduce error.

### Q.10 *Describe the significant figures. Also discuss its rules.*

**Ans.** SIGNIFICANT FIGURES

We know that physics is based on measurements whenever a physical quantity is measured; there is some uncertainty about its determined value. This uncertainty may be due to a number of reasons. One reason is the type of instrument, being used. We know that every measuring instrument is calibrated to a certain smallest division and this fact put a limit to the degree of accuracy while measuring with it.

Suppose that we want to measure the length of a straight line with the help of a meter rod calibrated in millimeters. Let the end point of the line lies between 10.3 and 10.4 cm marks. By convention if the end of line does not touch or cross the midpoint of the smallest division, the reading is confined to the previous division. In case the end of line seems to be touching or have crossed the midpoint, the reading is extended to the next division.

**For Your Information**

| | Interval (s) |
|---|---|
| Age of the universe | $5 \times 10^{17}$ |
| Age of the Earth | $1.4 \times 10^{17}$ |
| One year | $3.2 \times 10^{7}$ |
| One day | $8.6 \times 10^{4}$ |
| Time between normal heartbeats | $8 \times 10^{-1}$ |
| Period of audible sound waves | $1 \times 10^{-3}$ |
| Period of typical radio waves | $1 \times 10^{-6}$ |
| Period of vibration of an atom in a solid | $1 \times 10^{-13}$ |
| Period of visible light waves | $2 \times 10^{-15}$ |

**Approximate Values of Some Time Intervals**

By applying the above rule the position of the edge of line recorded as 12.7cm with the help of a meter rod calibrated in millimeters may lie between 12.65cm and 12.75cm. Thus in this example the maximum uncertainty is ±0.05 cm. It is, infact, equivalent to an uncertainty of 0.1cm equal to the least count of the instrument divided into two parts, half above and half below the recorded reading.

Thus the correct way of recording the above reading is

$$12.7 \pm 0.05 \text{ cm}$$

The recorded value of the length of the straight line i.e. 12.7 cm contains three digits (1, 2, 7) out of which two digits 1 and 2 are accurately known while the third digit i.e. 7 is a doubtful one. Thus significant figures may be stated as:

*"In any measurement, the accurately known digits and the first doubtful digit are called significant figures."* **(OR)** *"A significant figure is the one which is known to be reasonably reliable".*

If the above mentioned measurement is taken by a better measuring instrument which is exact up to hundredth of a centimeter, it would have been recorded as 12.70cm. In this case number of significant figures is four (1, 2, 7 and 0).

Thus, we can say that as we improve the quality of our measuring instrument and techniques, we extend the measured result to more and more significant figures and correspondingly improve the experimental accuracy of the result.

## General Rules

There are some general rules in order to find the number of significant figures in final result.

(1) All digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are significant. However zeros may or may not be significant. In case of zeros, the following rules may be adopted.

(a) A zero between two significant figures is itself significant.

(b) Zeros to the left of significant figures are not significant. For example, none of the zero in 0.00467 or 02.59 is significant.

(c) Zeros to the right of significant figure may or may not be significant.

(i) In decimal fraction, zeros to the right of a significant figure are significant. For example, all the zeros in 3.570 or 7.4000 are significant.

(ii) In integers, such as 8,000 kg, the number of significant zeros is determined by the accuracy of the measuring instrument. If the measuring scale has a least count of 1kg then there are four significant figures written in scientific notation as $8.000 \times 10^3$ kg. If the least count of the scale is 10 kg, then the number of significant figures will be 3 written in scientific notation as $8.00 \times 10^3$kg and so on.

(2) **Significant Figures in Multiplication and Division of Numbers**

In multiplying or dividing numbers i.e.,

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^{4}}{1.336} = 1.4576898 \times 10^{3}$$

Let us see, how many numbers should be retained in the answer. As the factor $3.64 \times 10^4$, the least accurate in the above calculations has three significant figures, the answer should be written to three significant figures only. The other figures are insignificant and should be deleted. While deleting the figures, the last significant figure to be retained is rounded off.

### Rules for Rounding Off Numbers

Following are the rules for rounding off numbers.

(a) If the first digit to be dropped is less than 5, the last digit to be retained should remain unchanged. e.g. 12.4 is rounded off as 12.

(b) If the first digit to be dropped is more than 5, the last digit to be retained is increased by one. e.g. 12.6 is rounded off as 13.

(c) If the digit to be droped is 5, and the number following 5 is not zero then the last digit to be retained is increased by one e.g. 12.51 is rounded off as 13.

(d) If the digit to be droped is 5, and the number following 5 is zero then the last digit to be retained follows odd even rule. i.e., if the digit to be retained is odd it is increased by one but left as it is if it is even.

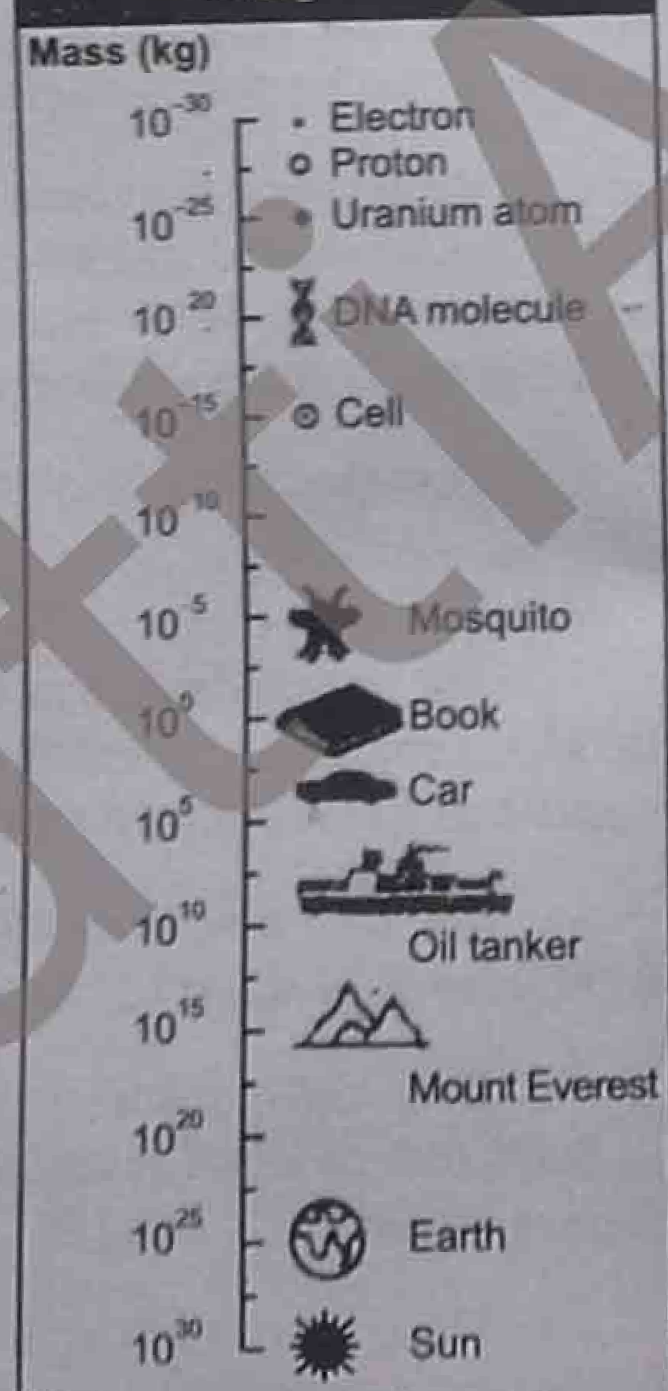
e.g.,

| | | |
|---|---|---|
| 13.50 | is rounded off as | 14 |
| 14.50 | is rounded off as | 14 |
| 15.5 | is rounded off as | 15 |
| 16.5 | is rounded off as | 16 |

### Examples of Numbers to be Rounded Off

The following numbers are rounded off to three significant figures as follows.

**Interesting Information**




Order of magnitude of some masses



| | | |
|---|---|---|
| 43.75 | is rounded off as | 43.8 |
| 56.8546 | is rounded off as | 56.9 |
| 73.650 | is rounded off as | 73.6 |
| 64.350 | is rounded off as | 64.4 |

Following this rule, the correct answer of the computation given in section (2) is $1.46 \times 10^3$.

(3) **Addition or Subtraction of Numbers**

In adding or subtracting numbers, the number of decimal places retained in the answer should be equal to the smallest number of decimal places in any of the quantities being added or subtracted. In this case, the number of significant figures is not important. It is the position of decimal that matters.

For example, we wish to add the following quantities expressed in meters.

| (i) | | (ii) |
|---|---|---|
| 72.1 | | 2.7543 |
| 3.42 | | 4.10 |
| 0.003 | | 1.273 |
| 75.523m | | 8.1273 |
| 75.5m | | 8.13m |

In Case (i) the number 72.1 has the smallest number of decimal places, thus answer is rounded off to the same decimal position which is then 75.5m.

In case (ii) the number 4.10 has the smallest number of decimal places, and hence answer is rounded off to the same decimal position which is then 8.13m.

**Do You Know?**

Mass can be thought of as a form of energy. In fact the mass is highly concentrated form of energy. Einstein's famous equation, $E = mc^2$ means

Energy = Mass x Speed of light$^2$

According to this equation 1 kg mass is actually $9 \times 10^{16}$ J energy.

***Q.11 Explain the term precision and accuracy.***

**Ans.** **PRECISION AND ACCURACY**

In measurements made in physics, the term precision and accuracy are frequently used. The precision of a measurement is determined by the instrument or device being used and accuracy of measurement depends on the fractional or percentage uncertainty in that measurement. Let us make it clear by examples.

### Example-I

Let the length of an object is recorded as 25.5cm by using meter rod having smallest division in millimeter. This measurement is difference of two readings that is initial and final positions. In case of single reading uncertainty is taken as ± 0.05cm. But in present reading uncertainty is taken double due to the reading of initial and final position i.e.

$$\text{Uncertainty} = \pm 0.05 \pm 0.05 = \pm 0.1\text{cm}$$

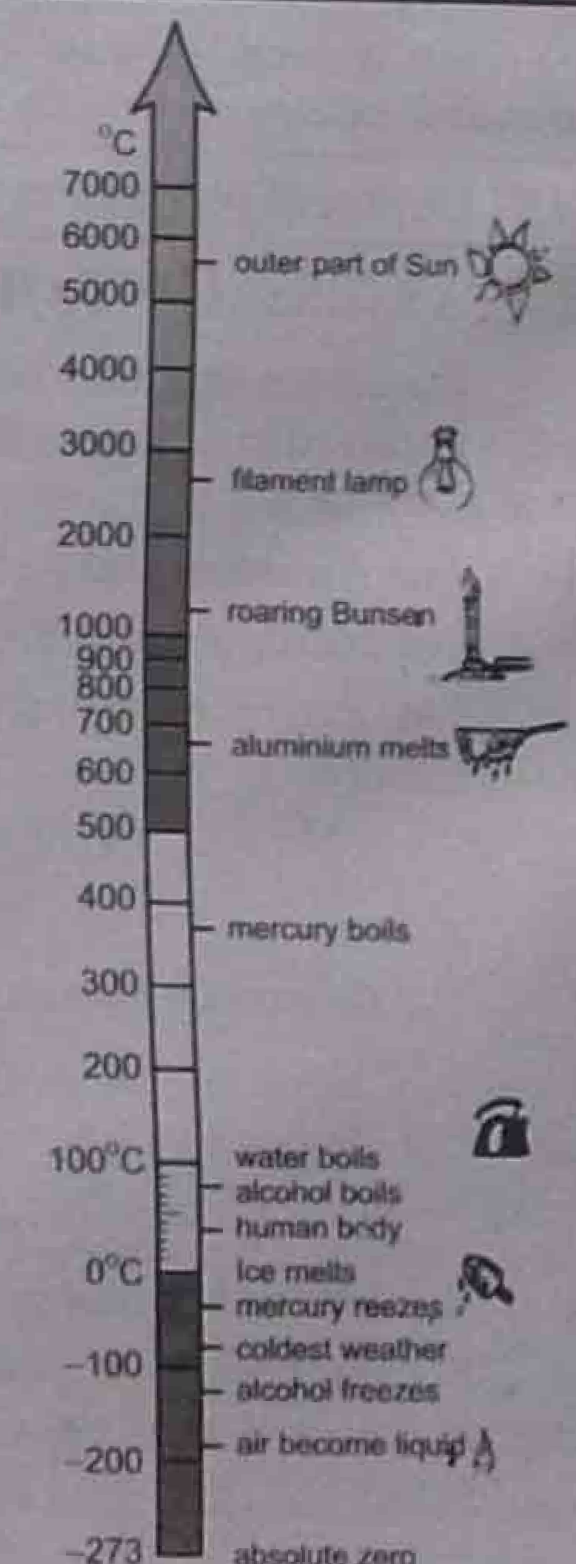
This uncertainty is called absolute uncertainty and absolute uncertainty in effect is equal to least count of the measuring instrument.

Thus,

Precision or absolute uncertainty (least count) = ±0.1cm

$$\text{Fractional uncertainty} = \frac{0.1\text{cm}}{25.5\text{cm}} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1}{25.5} \times \frac{100}{100} = \frac{0.4}{100} = 0.4\%$$




Some Specific Temperatures

**Example-II**

Another measurement is recorded as 0.45cm. It is taken by vernier callipers with least count as 0.01cm.

Now, Precision or absolute uncertainty (least count) = ±0.01cm

$$\text{Fractional uncertainty} = \frac{0.01\text{cm}}{0.45\text{cm}} = 0.02$$

$$\text{Percentage uncertainty} = \frac{0.01\text{cm}}{0.45\text{cm}} \times \frac{100}{100} = \frac{2.0}{100} = 2.0\%$$

**For Your Information**

We use many devices to measure physical quantities, such as length, time and temperature. They all have some limit of precision.

**Conclusion**

The reading 25.5cm taken by meter rule is although less precise but is more accurate having less percentage uncertainty or error where as the reading 0.45cm taken by vernier callipers is more precise but less accurate.

In fact, it is relative measurement which is important. The smaller a physical quantity, a more precise instrument should be used. Here the measurement 0.45cm demands that a more precise instrument, such as micrometer screw guage with least count 0.001cm should have been used. Thus we can define precision and accuracy.

**Precision**

A precise measurement is the one which has less absolute uncertainty.

**Accuracy**

An accurate measurement is the one which has less fractional or percentage uncertainty or error.

***Q.12 How will you assess the total uncertainty in the final result? Explain in different cases.***

**Ans. ASSESSMENT OF TOTAL UNCERTAINTY IN THE FINAL RESULT**

To assess the total uncertainty or error, it is necessary to evaluate the uncertainties in all the factors involved in that calculation. The maximum possible uncertainty or error in the final result can be found as follows.

**(1) For Addition and Subtraction**

For addition and subtraction absolute uncertainties are added.

For example, the distance x determined by the difference between two separate position measurements.

$$x_1 = 10.5 \pm 0.1\text{cm}$$

and

$$x_2 = 26.8 \pm 0.1\text{cm}$$

The difference between them is recorded as

$$x = x_2 - x_1$$

$$x = 26.8 \pm 0.1 - 10.5 \pm 0.1$$

$$x = 16.3 \pm 0.2\text{cm}$$

**(2) For Multiplication and Division**

For multiplication and division percentage uncertainties are added. For example, we determine maximum uncertainty in the value of resistance R of a conductor determined by formula.

$$R = \frac{V}{I}$$



Where V = Potential difference

and I = Current

The given values of V and I are.

$$V = 5.2 \pm 0.1\text{V}$$

and

$$I = 0.84 \pm 0.05\text{A}$$

$$\text{The \%age uncertainty for V is} = \frac{0.1\text{V}}{5.2\text{V}} \times \frac{100}{100} = \frac{2}{100} = 2\%.$$

$$\text{The \%age uncertainty for I is} = \frac{0.05\text{A}}{0.84\text{A}} \times \frac{100}{100} = \frac{6}{100} = 6\%$$

Hence total uncertainty in the value of R is.

= % age uncertainty for V + % age uncertainty for I.

= 2% + 6%

= 8%

Thus,

$$R = \frac{5.2\text{V}}{0.84\text{A}} = 6.19\ \text{VA}^{-1} = 6.19$$

Ohms with % age uncertainty of 8%.

Now,

$$8\% \text{ of } 6.2 = 6.2 \times \frac{8}{100} = 0.5$$

Thus,

$$R = 6.2 \pm 0.5 \text{ ohms}$$

The result is rounded off to two significant digits because both V and I have two significant figures. Uncertainty being an estimate only, is recorded by one significant figure.

**(3) For Power Factor**

For power factor multiply the percentage uncertainty by that factor i.e.

Total % age uncertainty = Power factor × % age uncertainy.

***Example***

Let us calculate the volume of a sphere given by formula.

$$V = \frac{4}{3}\pi r^3$$

Now,

% age uncertainty in V = power factor × % age uncertainty in r.

∴ % age uncertainty in V = 3 × % age uncertainty in r.

Let the radius of sphere is measured as 2.25 cm by a vernier calliper with least count 0.01 cm, then.

Radius r is recorded as

$$r = 2.25 \pm 0.01\text{cm}$$

Now,

Absolute uncertainty = least count = 0.01cm

$$\text{\% age uncertainty in r} = \frac{0.01\text{cm}}{2.25\text{cm}} \times \frac{100}{100} = \frac{0.4}{100} = 0.4\%$$

∴ Total % age uncertainty in V = 3 × 0.4% = 1.2%

**For Your Information**

Colour printing uses just four colours-cyan, magenta, yellow and black to produce the entire range of colours. All the colours in this book have been made from just these four colours.

Now,

$$\text{Volume} = V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}(3.14)(2.25\text{cm})^3$$

$$= 47.689\ \text{cm}^3$$

Thus,

Volume V = 47.689 $\text{cm}^3$ with 1.2% of uncertainty.

As, 1.2% of 47.689 = 0.6

$\therefore$ $V = 47.7 \pm 0.6\ \text{cm}^3$

**(4) For Uncertainty in the Average Value of Many Measurements**

(i) Find the average value of measured values.

(ii) Find the deviation of each measured value from the average value.

(iii) The mean deviation is the uncertainty in the average value.

***Example***

There are six readings of the micrometer screw guage to measure the diameter of a wire in mm.

The readings are.

1.20, 1.22, 1.23, 1.19, 1.22, and 1.21.

Then,

$$\text{Average} = \frac{1.20 + 1.22 + 1.23 + 1.19 + 1.22 + 1.21}{6}$$

$$= 1.21\ \text{mm}$$

The deviation of each value ie the difference between each recording and average value, without regard of sign, are.

0.01, 0.01, 0.02, 0.02, 0.01, and 0.

$$\text{Mean of deviations} = \frac{0.01 + 0.01 + 0.02 + 0.02 + 0.01 + 0}{6}$$

$$= 0.01\ \text{mm}$$

Thus uncertainty in mean diameter i.e., 1.21 mm is 0.01 mm recorded as

$$\boxed{\text{Diameter} = 1.21 \pm 0.01\ \text{mm}}$$

**(5) For the Uncertainty in Timing Experiment**

The uncertainty in the time period of a vibrating body is found by dividing the least count of timing device by the number of vibrations ie.

$$\text{Uncertainty in time period} = \frac{\text{Least count}}{\text{No. of vibrations}}$$

**For Your Information**

These are not decoration pieces of glass but are the earliest known exquisite and sensitive thermometers, built by the Academia del Cimento (1657-1667), in Florence. They contained alcohol, some times coloured red for easier reading.



***Example***

The time of 30 vibrations of a simple pendulum recorded by a stopwatch accurate up to one tenth of a second (least count) is 54.6 s.

$$\text{Now, Time period} = T = \frac{54.6\ \text{s}}{30} = 1.82\ \text{Sec}$$

$$\text{Uncertainty} = \frac{\text{Least count}}{\text{No. of vibrations}} = \frac{0.1\ \text{s}}{30} = 0.003\text{S}$$

Thus time period T is

$$\boxed{T = 1.82 \pm 0.003\ \text{S}}$$

**Note:** *It is advisable to count large number of swings to reduce timing uncertainty.*

**For Your Information**

Atomic Clock

The cesium atomic frequency standard at the National Institute of Standards and Technology in Colorado (USA). It is the primary standard for the unit of time.

### *Q.13 What do you understand from the dimensions of physical quantities?*

**Ans DIMENSIONS OF PHYSICAL QUANTITIES**

*"The qualitative nature of the physical quantity is considered a dimensions. It is denoted by a specific symbol written within square brackets."*

**(OR)**

Dimensions of physical quantity is a relationship between derived physical quantity and the base quantity.

Different quantities such as length, breadth, diameter, light year which are measured in meter denote the same dimensions and has the dimensions of length [L]. Similarly, the dimensions of mass and time are denoted by [M] and [T] respectively.

Other quantities that we measure have dimensions which are combinations of these dimensions.

**(1) Dimensions of Speed**

$$\text{Speed} = \frac{\text{length}}{\text{time}} = \frac{L}{T}$$

$$\text{Dimensions of speed} = \frac{\text{dimension of length}}{\text{dimension of time}}$$

$$\text{or} \quad [V] = \frac{[L]}{[T]}$$

$$\Rightarrow \quad [V] = [L][T^{-1}] = [LT^{-1}]$$

**(2) Dimensions of Acceleration**

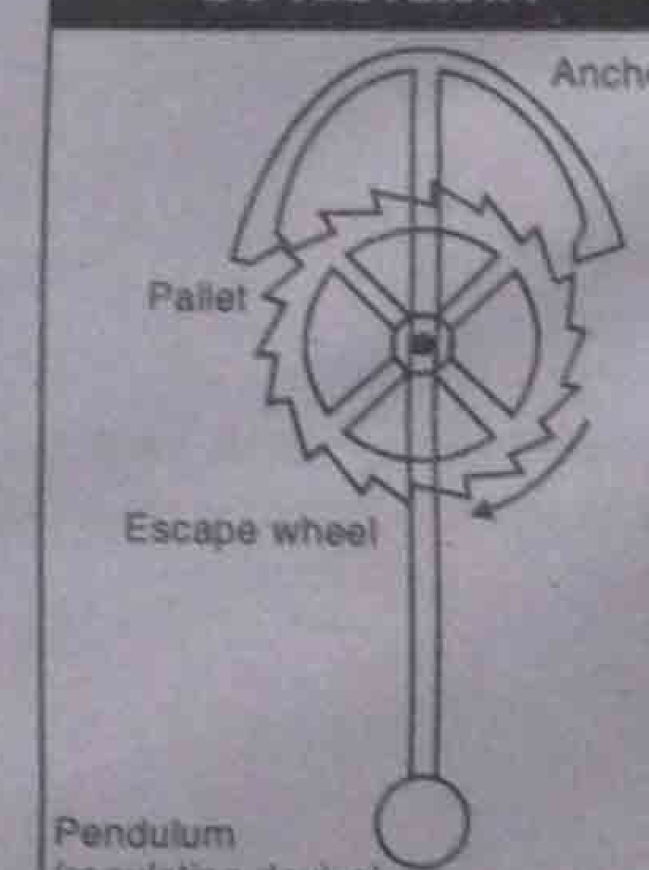
$$\text{Acceleration} = \frac{\text{velocity}}{\text{time}}$$

$$\text{Dimensions of acceleration} = \frac{\text{dimension of V}}{\text{dimension of T}}$$

$$[a] = \frac{[V]}{[T]}$$

$$[a] = \frac{[LT^{-1}]}{[T]} = [LT^{-1}][T^{-1}] = [LT^{-2}]$$

**Do You Know?**




The device which made the pendulum clock practical.

(3) **Dimensions of Force**

Since, $F = ma$

$\therefore$ $[F] = [m]\,[a]$

$= [M]\,[LT^{-2}] \qquad \therefore [a] = [LT^{-2}]$

$= [MLT^{-2}]$

Using the method of dimensions called the dimensional analysis, we can check the correctness of a given formula or an equation and can also derive it. Dimensional analysis makes use of the fact that expression of the dimensions can be manipulated as algebraic quantities.

(i) ***Checking the Homogeneity of Physical Equation***

In order to check the correctness of an equation, we are to show that the dimensions of the quantities on both sides of the equation are the same, irrespective of the form of the formula. This is called the principle of homogeneity of dimensions.

(ii) ***Derivation of a Possible Formula***

The success of this method for deriving a relation for a physical quantity depends on the correct guessing of various factors on which the physical quantity depends.



# SOLVED EXAMPLES

***EXAMPLE 1.1***

**The length, breadth and thickness of a sheet are 3.233m, 2.105 m and 1.05 cm respectively. Calculate the volume of the sheet correct upto the appropriate significant digits.**

***Data***

Length of sheet $= l = 3.233\text{m}$

Breadth of sheet $= b = 2.105\text{m}$

Thickness of sheet $= h = 1.05\text{cm} = 1.05 \times 10^{-2}\text{m}$

***To Find***

Volume of sheet $= V = ?$

***SOLUTION***

Volume is given by

$$V = l \times b \times h$$

Putting values, we get

$$V = 3.233 \times 2.105 \times 1.05 \times 10^{-2}$$

$$= 7.14573825 \times 10^{-2}\ \text{m}^3$$

As the factor 1.05 cm has minimum number of significant figures equal to three, therefore, volume is recorded up to 3 significant figures.

Hence, $\boxed{V = 7.15\ \text{m}^3}$

***Result***

Volume of sheet $= V = 7.15\ \text{m}^3$

***EXAMPLE 1.2***

**The mass of a metal box measured by a lever balance is 2.2 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct upto the appropriate precision?**

***Data***

Mass of box $= m = 2.2\ \text{kg}$

Mass of $1^{st}$ coin $= m_1 = 10.01\text{g} = 0.01001\text{kg}$

Mass of $2^{nd}$ coin $= m_2 = 10.02\text{g} = 0.01002\text{kg}$

***To Find***

Total mass of box $= M = ?$

**SOLUTION**

Total mass when silver coins are added.

$$M = m + m_1 + m_2$$
$$= 2.2 + 0.01001 + 0.01002$$
$$= 2.22003 \text{ kg}$$

Since least precise is 2.2 kg having one decimal Place, hence total mass should be to one decimal place which is the appropriate precision. Thus,

Total mass = 2.2 kg

***Result***

Total mass = 2.2 kg

**EXAMPLE 1.3**

**The diameter and length of a metal cylinder measured with the help of vernier calipers of least count 0.01 cm are 1.22 cm and 5.35 cm. Calculate the volume V of the cylinder and uncertainty in it.**

***Data***

Least count of vernier callipers = 0.01cm
Diameter of cylinder = D = 1.22 cm
Length of cylinder = $l$ = 5.35 cm

***To Find***

Volume of cylinder = V = ?
Uncertainty in volume = ?

**SOLUTION**

Absolute uncertainty in length = 0.01 cm

% age uncertainty in length $= \frac{0.01}{5.33} \times \frac{100}{100} = \frac{0.2}{100} = 0.2\%$

Absolute uncertainty in diameter = 0.01 cm

% uncertainty in diameter $= \frac{0.01}{1.22} \times \frac{100}{100} = \frac{0.8}{100} = 0.8\%$

As volume is given by

$$V = \pi r^2 l$$
$$V = \frac{\pi d^2 l}{4}$$

∴ Total uncertainty in V = 2 (% uncertainty in d) + (% uncertainty in $l$)
= 2 (.8) + .2
= 1.6 + 2 = 1.8%

Now, $V = \frac{(3.14)\,(1.22)^2\,(5.33)}{4} = 6.2509079 \text{ cm}^3$



Thus, V = 6.2509079 $cm^3$ with uncertainty 1.8%.
As, 1.8% of 6.2509079 = 0.1
Thus, V = 6.2 ± 0.1 $cm^3$
Where 6.2 $cm^3$ is calculated volume and 0.1 $cm^3$ is the uncertainty in it.

***Result***

Volume of cylinder = 6.2509079 $cm^3$
Uncertainty in volume = 6.2 ± 0.1 $cm^3$

**EXAMPLE 1.4**

**Check the correctness of the relation $v = \sqrt{\frac{F \times l}{m}}$ where V is the speed of transverse wave on a stretched string of tension F, length $l$ and mass m.**

**SOLUTION**

The given formula is

$$V = \sqrt{\frac{F \times l}{m}}$$

Dimensions of L.H.S = [V] = $[LT^{-1}]$

Dimensions of R.H.S $= \left[\frac{F \times l}{m}\right]^{1/2}$

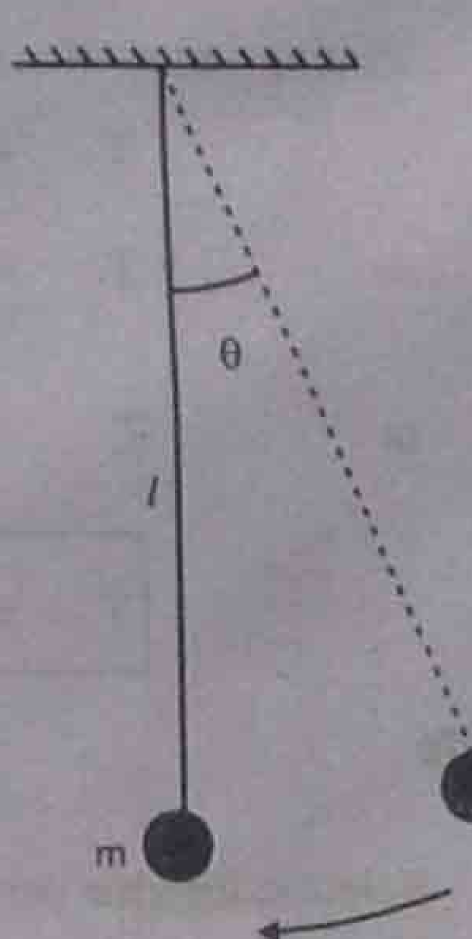
$$= \left[\frac{[F]\,[l]}{[m]}\right]^{1/2} = \left[\frac{[MLT^{-2}]\,[L]}{[M]}\right]^{1/2}$$
$$= [L^2T^{-2}]^{1/2}$$
$$= [LT^{-1}]$$

As Dimensions of LHS = Dimensions of RHS.
Hence formula is dimensionally correct.

**EXAMPLE 1.5**

**Derive a relation for the time period of a simple pendulum (Fig. 1.2) using dimensonsal analysis. The various possible factors on which the time period T may depend are**

**(i) Length of the pendulum ($l$).**
**(ii) Mass of the bob (m).**
**(iii) Angle θ which the thread makes with the vertical.**
**(iv) Acceleration due to gravity (g).**




***Data***

Length of the pendulum = $l$
Mass of bob = m
Angle which the thread makes with vertical = θ
Acceleration due to gravity = g

***To Find***

Relation for time period = T = ?

**SOLUTION**

The relation for time period T will be of the form

$$T \propto m^a \times l^b \times \theta^c \times g^d$$

or $$T = \text{Constant } m^a l^b \theta^c g^d \quad \ldots\ldots (1)$$

Now we find the values of powers a, b, c and d.

Now, Dimensions of θ = $[LL^{-1}]$ = 1 $\quad \therefore s = r\theta$

And, Dimensions of g = $[LT^{-2}]$ $\quad \theta = \frac{S}{r} = \frac{L}{L}$

$$= LL^{-1} = 1$$

Writing dimensions on both sides, we get.

$$[T] = \text{Constant } [M]^a [L]^b [1]^c [LT^{-2}]^d$$
$$= \text{Constant } [M]^a [L]^b [1]^c [L]^d [T]^{-2d}$$
$$[T] = \text{Constant } [M]^a [L]^{b+d} [T]^{-2d} \quad ([1]^c = 1)$$

Comparing the exponents of M, L and T on both sides

$$[M]^o = [M]^a \Rightarrow a = 0$$
$$[L]^o = [L]^{b+d} \Rightarrow b + d = 0 \text{ or } b = -d = (-1/2) = ½$$
$$[T]^1 = [T]^{-2d} \Rightarrow -2d = 1 \text{ or } d = -1/2$$

Thus, a = 0

b = 1/2

d = −1/2

Putting values of a, b, d and θ in eq. (1), we get.

$$T = \text{Constant } m^o \times l^{1/2} \times 1 \times g^{-1/2}$$

or $$T = \text{Constant} \frac{l^{1/2}}{g^{1/2}}$$

or $$T = \text{Constant} \sqrt{\frac{l}{g}}$$

$$\boxed{T = \text{Constant} \sqrt{\frac{l}{g}}}$$

***Result***

Relation for time period = $T = \text{Constant} \sqrt{\frac{l}{g}}$



**EXAMPLE 1.6**

**Find the dimensions and hence, the SI units of coefficient of viscosity η in the relation of Stokes law for the drag force F for a spherical object of radius r moving with velocity v given as F = 6 πηrv.**

***Data***

$$F = 6\pi\eta rv$$

***To Find***

Dimensions of η = ?

Units of η = ?

**SOLUTION**

We are given

$$F = 6\pi\eta rv \quad \ldots\ldots (1)$$

Now, 6π is a number having no dimensions. It is not accounted in dimensional analysis, then.

$$[F] = [\eta r v]$$
$$\Rightarrow [\eta] = \frac{[F]}{[r][v]}$$

Substituting (putting) the dimensions of F, r and v in R.H.S, we get

$$[\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]}$$
$$[\eta] = [ML^{-1}T^{-1}]$$

**Units of η**

SI unit of η are $kg\,m^{-1}s^{-1}$.

***Result***

Dimension of η = $[ML^{-1}T^{-1}]$

Unit of η = $kg\,m^{-1}s^{-1}$

# SHORT QUESTIONS

**1.1 Name several repetitive phenomenon occurring in nature which could serve as reasonable time standard.**

**Ans.** Any natural phenomenon that repeats itself after exactly same time interval can be used as a measure of time. The repetitive phenomenon could serve as reasonable time standard, occurring in nature are as follows:

1. **Sun:** Sun served as reasonable time standard because sunset and sunrises gives the information of time.
2. **Moon:** Moon is also reasonable time standard because it gives the information of time.
3. **Weather:** Changing of weather can also give information about time.
4. Rotation of Earth on its axis.
5. Rotation of Earth around the sun.
6. Oscillation of a simple pendulum. , Heart beat, pulse rate

**1.2 Give draw backs to use the period of a pendulum as a time standard.**

**Ans.** As we know that the time period of a simple pendulum depends upon the length and value of g at any place. Since

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(i) It is clear that time period of a simple pendulum depends upon the value of g which is different at different places. So a pendulum of same length may have different time period at difference places. So period of pendulum cannot be taken as standard for measuring time.

(ii) **Friction:** Time period of a simple pendulum changes due to air resistance.

(iii) **Temperature:** In summer due to increase in temperature, length of simple pendulum changes so time period changes.

**1.3 Why do we find it useful to have two units for the amount of substance, the kilogram and the mole?**

**Ans.** It is very useful to have two units for the amount of substance i.e., kilogram and mole. If we want to consider a specific amounts of mass without considering number of microscopic atoms present in it, it is useful to use **kilogram**. Because one kilogram of different substances contains different number of molecules. While if we want to consider a fixed number of atoms present in it then it is useful to use **mole**. Because one mole of any substance contains the same number of atoms or molecules.

**1.4 Three student's measured the length of a needle with a scale on which minimum division is 1 mm and recorded as (i) 0.2145m (ii) 0.21m (iii) 0.214m. Which record is correct and why?**

**Ans.** In these records (iii) 0.214 m is more correct than the other records because the least count of a scale is 1 mm which can be written as 0.001 m. So according to this figure, the student measure that type of record is correct.



**1.5 An old saying is that "A chain is only as strong as its weakest link". What analogue statement can you make regarding experimental data used in computation?**

the result of an experiment is only as accurate as its least accurate reading is used in experiment

**Ans.** The analogous statement regarding experimental data used in computation is "A result obtained from an experimental data used in computation is only as accurate as its least accurate reading".

**1.6 The period of simple pendulum is measured by a stopwatch. What type of errors are possible in the time period?**

**Ans.** When the period of a simple pendulum is measured by a stopwatch, the following types of errors are possible:

1. **Systematic Error:** The error due to the fault in the measuring instrument is called systematic error i.e., zero error, poor calibration
2. **Personal Error:** The error due to the faulty procedure of an observer is called personal error.

**1.7 Does dimensional analysis give any information on constant of proportionality that may appear in an algebraic expression? Explain.**

**Ans.** Dimensional analysis does not give any information about the constant of proportionality or dimensionless constant. For example

$$v = \text{Constant} \times \sqrt{\frac{E}{\rho}}$$

The numerical value of this constant cannot determined by dimensional analysis.

**1.8 Write the dimension of:**

(i) **Pressure** (ii) **Density**

**Ans.** (i) **Dimensions of Pressure:**

$$\text{As} \quad P = \frac{F}{A} = \frac{ma}{A}$$

$$\text{Unit of P} = \frac{kg\,ms^{-2}}{m^2} = kg\,m^{-1}s^{-2}$$

$$\Rightarrow [P] = [ML^{-1}T^{-2}]$$

(ii) **Dimensions of Density:**

$$\text{As} \quad \text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Unit of density} = \frac{kg}{m^3} = kg\,m^{-3}$$

$$[\text{Density}] = [ML^{-3}]$$

**1.9 The wavelength λ of a wave depends on the speed v of the wave and its frequency f. Knowing that:**

$[\lambda] = [L], [v] = [LT^{-1}]$ and $[f] = [T^{-1}]$

**Decide which of the following is correct, $f = v\lambda$ or $f = \frac{v}{\lambda}$.**

**Ans.** In 1[st] case if $f = v\lambda$ where f is frequency. Its dimension is $[T^{-1}]$, v is speed, its dimensions are $[LT^{-1}]$.

$\lambda$ is the wavelength, its dimension is [ L ].

So,
$$[T^{-1}] = [LT^{-1}][L]$$
$$[T^{-1}] = [L^2T^{-1}]$$

Hence the equation $f = v\lambda$ is not dimensionally correct because left hand side dimension is not equal to right hand side dimension.

In second case

$$f = \frac{v}{\lambda}$$

So
$$[T^{-1}] = \frac{[LT^{-1}]}{[L]}$$
$$[T^{-1}] = [T^{-1}]$$

Hence the equation $f = \frac{v}{\lambda}$ is dimensionally correct because left hand side dimensions is equal to right hand side dimension.



# PROBLEMS WITH SOLUTIONS

## PROBLEMS 1.1

**A light year is the distance light travels in one year. How many metres are there in one light year? (Speed of light = $3.0 \times 10^8$ ms$^{-1}$)**

***Data***

Time = t = 1 year
= 365 days
= $365 \times 24 \times 3600$
= 31536000 sec.

Speed of light = C = $3 \times 10^8$ m/s

***To Find***

Distance covered by light = d = ?

## SOLUTION

As we know that

$$v = \frac{d}{t}$$

$$d = v \times t$$
$$= C \times t$$
$$= 3 \times 10^8 \times 31536000$$
$$= 94608000 \times 10^8$$
$$= 9.46 \times 10^{15} \text{ m}$$

***Result***

Distance covered by light = d = $9.46 \times 10^{15}$ m

## PROBLEM 1.2

**(a) How many seconds are there in 1 year?**

**(b) How many nanoseconds in 1 year?**

**(c) How many years in 1 second?**

***Data***

One year = 365 days
= $365 \times 24 \times 3600$
= 3.1536000
= $3.15 \times 10^7$ sec.

### To Find

(a) Seconds in one year = ?

(b) Nanosecond in one year = ?

(c) Years in one second = ?

### SOLUTION

(a) As we know that

$$1 \text{ year} = 365 \text{ days}$$
$$= 365 \times 24 \times 3600$$
$$= 3.15 \times 10^7 \text{ sec.}$$

Seconds in one year $= 3.15 \times 10^7$ sec.

(b) As 1 year $= 3.15 \times 10^7 \times 10^9$ n sec. Since 1 ns $= 10^{-9}$ s

$= 3.15 \times 10^{16}$ nanosecond $\therefore$ 1 s $= 10^9$ ns

(c) As 1 year $= 3.15 \times 10^7$ sec.

$$\frac{1}{3.15 \times 10^7} \text{ year} = 1 \text{ second}$$

$$1 \text{ second} = 0.317 \times 10^{-7} \text{ years}$$
$$= 3.17 \times 10^{-8} \text{ years}$$

### Result

(a) Number of seconds in one year $= 3.15 \times 10^7$ seconds

(b) Number of nanoseconds in one year $= 3.15 \times 10^{16}$ nanosecond

(c) Number of years in one second $= 3.17 \times 10^{-8}$ years

## PROBLEM 1.3

**The length and width of a rectangular plate are measured to be 15.3 cm and 12.80 cm, respectively. Find the area of the plate.**

### Data

Length of rectangular plate = L = 15.3 cm

Width of rectangular plate = W = 12.80 cm

### To Find

Area of the plate = A = ?

### SOLUTION

As we know that

$$\text{Area} = \text{Length} \times \text{Width}$$
$$= 15.3 \times 12.80$$
$$= 195.84 \text{ cm}^2$$

### Result

Area of rectangular plate = A = 196 $\text{cm}^2$



## PROBLEM 1.4

**Add the following masses given in kg upto appropriate precision. 2.189, 0.089, 11.8 and 5.32.**

### Data

The given masses are 2.189, 0.089, 11.8 and 5.32

### To Find

Sum of masses upto appropriate precision = ?

### SOLUTION

$$\text{Sum of masses} = 2.189 + 0.089 + 11.8 + 5.32$$
$$= 19.398$$
$$= 19.4 \text{ kg}$$

### Result

Sum of masses upto appropriate precision = 19.4 kg

## PROBLEM 1.5

**Find the value of 'g' and its uncertainty using $T = 2\pi\sqrt{\frac{l}{g}}$ from the following measurements made during an experiment.**

**Length of simple pendulum $l$ = 100 cm**

**Time for 20 vibrations = 40.2 s**

**Length was measured by a metre scale of accuracy upto 1 mm and time by stop watch of accuracy upto 0.1 s.**

### Data

Length of simple pendulum = $l$ = 100 cm

= 1 m

Time for 20 vibration = t = 40.2 s

Time period = $T = \frac{t}{20} = \frac{40.2}{20} = 2.01$ sec.

Least count of metre scale = 1 mm = 0.1 cm

### To Find

Acceleration due to gravity = g = ?

### SOLUTION

As we know that

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring

$$T^2 = 4\pi^2 \times \frac{l}{g}$$

$$\boxed{g = \frac{4\pi^2 l}{T^2}}$$

Putting the values

$$g = \frac{4(3.14)^2 \times 1}{(2.01)^2}$$

$$= \frac{39.4384}{4.04}$$

$$g = 9.76\ m/s^2$$

Since Uncertainty in length = 0.1 cm

% uncertainty in length = 0.1%

% uncertainty in time $= \frac{0.005}{2.01} \times 100$  Since $\frac{0.1}{20} = 0.005$ sec.

= 0.25%

Thus Total uncertainty in "g" = % uncertainty in time + 2(% uncertainty in time)

= 0.1 + 2(0.25)

= 0.1 + 0.5

= 0.6%

Thus Uncertainty in calculated value of g $= \frac{0.6}{100} \times 0.97$

$= 0.06\ m/s^2$

Hence $g = (9.76 \pm 0.06)\ m/s^2$

***Result***

Acceleration due to gravity $= g = (9.76 \pm 0.06)\ m/s^2$

***PROBLEM 1.6***

**What are the dimensions and units of gravitational constant G in the formula?**

$$F = G\frac{m_1 m_2}{r^2}$$

***Data***

The given formula is

$$F = G\frac{m_1 m_2}{r^2}$$

***To Find***

Dimensions of G = ?

Unit of G = ?

***SOLUTION***

Now for dimensions

$$F = G\frac{m_1 m_2}{r^2}$$



$$G = \frac{F \times r^2}{m_1 m_2} \quad \text{Since } F = ma$$

$$= \frac{ma \times r^2}{m_1 \times m_2}$$

$$\text{Unit of } G = \frac{kg\ .\ m/s^2 \times m^2}{kg\ .\ kg}$$

$$= \frac{m^3}{kg\ s^2}$$

Dimensions of G $= [M^{-1}L^3T^{-2}]$

For unit of G

$$G = \frac{F \times r_2}{m_1 \times m_1} = \frac{N\text{-}m^2}{kg\ .\ kg}$$

$$G = Nm^2/kg^2$$

***Result***

Dimensions of G $= [M^{-1}L^3T^{-2}]$

Unit of G $= N\text{-}m^2/kg^2$

***PROBLEM 1.7***

**Show that the expression $v_f = v_i + at$ dimensionally correct, where $v_1$ is the velocity at $t = 0$, a is acceleration and $v_f$ is the velocity at time t.**

***Data***

The given equation is

$$v_f = v_i + at$$

***To Find***

Is the equation dimensionally correct = ?

***SOLUTION***

Now $v_f = v_i + at$

In unit form

$$\frac{m}{s} = \frac{m}{s} + \frac{m}{s^2} \times s$$

$$\frac{m}{s} = \frac{m}{s} + \frac{m}{s}$$

$$\frac{L}{T} = \frac{L}{T} + \frac{L}{T}$$

Where 2 is constant so it is dimensionless

$$\frac{L}{T} = \frac{L}{T}$$

$$[LT^{-1}] = [LT^{-1}]$$

$$[LT^{-1}] = [LT^{-1}]$$

***Result***

Hence the equation $v_f = v_i + at$ is dimensionally correct.

**PROBLEM 1.8**

**The speed v of sound waves through a medium may be assumed to depend on (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to strain. Deduce by the method of dimensions, the formula for the speed of sound.**

**SOLUTION**

As we know that the speed of sound depends upon the following two factors

(i) Density $\rho^a$ and (ii) Elasticity $E^b$

Since $v \propto \rho^a E^b$

$$v = \text{Constant} \times \rho^a E^b \quad \text{...... (i)}$$

Writing dimensions of quantities on both the sides.

Dimensions of velocity $v = \left[\frac{S}{T}\right] = [LT^{-1}]$

The dimensions of density $\rho = \frac{\text{Mass}}{\text{Volume}}$

$$\text{Unit of } \rho = \frac{kg}{m^3}$$

$$[\rho] = [ML^{-3}]$$

and Dimensions of elasticity $E = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A}$

Where strain has no dimensions

$$E = \frac{ma}{A}$$

$$\text{Unit of E} = \frac{kg\ m/s^2}{m^2}$$

$$[E] = \frac{kg}{m.s^2} = [ML^{-1}T^{-2}]$$

Putting in equation (i)

$$[LT^{-1}] = \text{Constant } [ML^{-3}]^a [ML^{-1}T^{-2}]^b$$

$$[LT^{-1}] = \text{Constant} \times [M^aL^{-3a}][M^bL^{-b}T^{-2b}]$$

$$= \text{Constant} \times [M^{a+b} L^{-3a-b} T^{-2b}]$$

Comparing the exponents

For L $\quad -3a - b = 1$

For T $\quad -2b = -1$

For M $\quad a + b = 0$

As $\quad -2b = -1$

$$b = \frac{1}{2}$$

and $\quad a + b = 0$

$$a + \frac{1}{2} = 0$$

$$\boxed{a = -\frac{1}{2}}$$

Putting the values in eq. (i)

$$v = \text{Constant} \times \rho^{-1/2} E^{1/2}$$

$$v = \text{Constant} \times \frac{E^{1/2}}{\rho^{1/2}}$$

$$v = \text{Constant} \times \sqrt{\frac{E}{\rho}}$$

***Result***

The formula for the speed of sound is

$$v = \text{Constant} \times \sqrt{\frac{E}{\rho}}$$

**PROBLEM 1.9**

**Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.**

**SOLUTION**

The given equation is

$$E = mc^2$$

Writing the dimension of both sides

Dimension of energy (E) = Work $= F . d$

$$= ma . d$$

$$\text{Unit of work} = kg \frac{m}{s^2} . m$$

$$= kg \frac{m^2}{s^2}$$

$$[W] = [ML^2T^{-2}] \quad \text{...... (i)}$$

$$\text{Unit of } mc^2 = kg\left(\frac{m}{s}\right)^2$$

$$= kg \frac{m^2}{s^2}$$

$$[mc^2] = [ML^2T^{-2}] \quad \text{...... (ii)}$$

From eq. (i) and (ii)

$$[ML^2T^{-2}] = [ML^2T^{-2}]$$

***Result***

Hence the Einstein's equation $E = mc^2$ is dimensionally consistent.

**PROBLEM 1.10**

**Suppose, we are told that the acceleration of a particle moving in a circle of radius r with uniform speed v is proportional to some power of r, say $r^n$, and some power of v, say $v^m$, determine the powers of r and v?**

**SOLUTION**

According to statement, the acceleration of particle moving in a circle can be written as

$$a \propto r^n v^m$$

$$a = \text{Constant} \times r^n v^m \qquad \text{...... (i)}$$

Writing the dimension of both sides

Dimensions of acceleration $= [a] = [LT^{-2}]$

Dimensions of radius $= [r] = m = [L]$

Dimensions of velocity $= [v] = [LT^{-1}]$

Putting in eq. (i)

$$[LT^{-2}] = \text{Constant} \times [L]^n [LT^{-1}]^m$$

$$[LT^{-2}] = \text{Constant} \times [L^n][L^m T^{-m}]$$

$$[LT^{-2}] = \text{Constant} \times [L^{n+m} T^{-m}]$$

Comparing the exponents

$$n + m = 1$$

$$-m = -2$$

$$\boxed{m = 2}$$

Putting in above

$$n + 2 = 1$$

$$n = 1 - 2$$

$$\boxed{n = -1}$$

Putting in eq. (i)

$$a = \text{Constant} \times r^{-1} v^2$$

$$a = \text{Constant} \times \frac{v^2}{r}$$

***Result***

The acceleration of a particle moves with velocity in a circle of radius r is

$$a = \text{Constant} \times \frac{v^2}{r}$$

# Chapter 2 VECTORS AND EQUILIBRIUM

**LEARNING OBJECTIVES**

**At the end of this chapter the students will be able to:**

- ★ Understand the definition of scalars and vectors.
- ★ Understand and use rectangular coordinate system.
- ★ Understand the idea of unit vector, null vector and position vector.
- ★ Represent a vector as two perpendicular components (rectangular components).
- ★ Understand multiplication of vectors and solve problems.
- ★ Define the moment of force or torque.
- ★ Appreciate the use of the torque due to a force.
- ★ Appreciate the applications of the principle of moments.

***Q.1 Define scalars and vectors. Give examples.***

**Ans.** **SCALAR QUANTITIES**

Those physical quantities which are completely described by magnitude with proper units are called scalar quantities. e.g. time, current, speed etc. Scalars are added, subtracted, divided and multiplied by ordinary arithmetic rules.

**VECTOR QUANTITIES**

Those physical quantities which are completely described by magnitude with proper units as well as direction are called vector quantities. e.g. force, torque etc. Vectors are not added, subtracted, divided and multiplied by ordinary arithmetic rules but it can be used as vector addition, vector multiplication and vector subtraction.

***Q.2 Describe how a vector quantity is represented?***

**Ans.** **REPRESENTATION OF A VECTOR**

(i) **By Letter**

A vector is usually represented by a bold face letter that is **A** or by a letter with an arrow drawn above or below it that is $\vec{A}$ or $\underline{A}$.

The magnitude of a vector is denoted by $|\vec{A}|$ (modulus) or A.